Grade 4

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 3

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

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Contents

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 7 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of "saying it, showing it and symbolising it". This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

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What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "say it" or auditory mode.
- The pictures and diagrams provide suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year’s work as required.

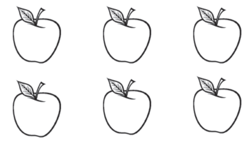
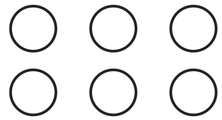
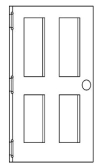
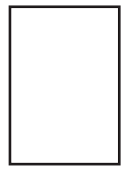
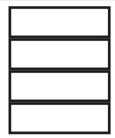
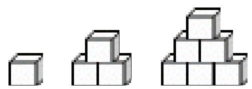
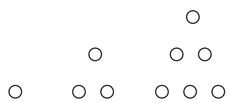
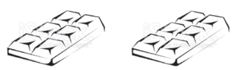
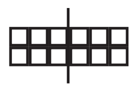
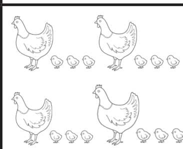
Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

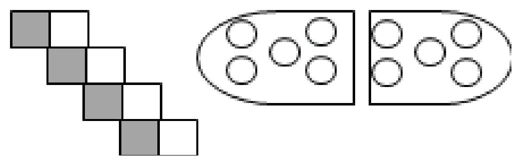
- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

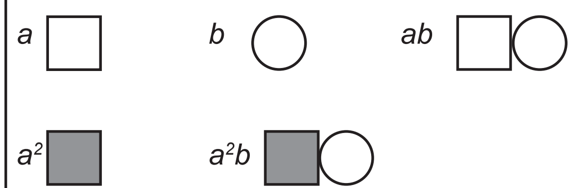
CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160 = 550cm$ Area: $L \times W = 195 \times 80 = 15\,600cm^2 = 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml = 1$ litre or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			1; 3; 6…	$\frac{n\,(n+1)}{2}$
Fraction	Chocolate bar			$\frac{6}{12}$	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of $12 = 6$
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	$500g = 0{,}5$ kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept	SHOW IT: to embody the idea	SYMBOL IT: to enable mathematising
	• Essential for introducing terminology in context • Supports learning through the auditory pathway • Important to link mathematics to everyday realities	• Essential to assist <u>storing and retrieving</u> concepts • Supports understanding through the <u>visual</u> pathway • Important to <u>condense</u> a variety of information into a single image	• Essential to assist <u>mathematical thinking</u> about concepts • Supports the <u>transition</u> from situations to mathematics • Important to <u>expedite</u> calculation and problem solving
FP: Doubling and halving	"To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use 2½ cups of flour, it means she has to use twice as much this time." "To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro's in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half."	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example:  Double Halve  3. Diagrams: 	$7 + 7 = \square$ $7 + \square = 14$ $\triangle + \bigcirc = 14$, but $\square + \square = 14$ 2 times 7 = 14 double of 7 is 14 $14 - 7 =$ $14 - \square = 7$ 14 divided by 2 = 7 14 halved is 7

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?"	o o oo o oo ooo Draw the next term in this pattern. T1 T2 T3 T4 o o oo o oo ooo o oo ooo oooo Describe this pattern. What is the value of the 9th term of this pattern (T9)? o o oo o oo ooo o oo ooo oooo o oo ooo oooo ooooo To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: 1; 3; 6... 1; 3; 6; 10... 1; 3; 6;10;15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.

SP: Grouping the terms of an algebraic expression	*“We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:”*	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagrammatic form 	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= - 3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: CAPACITY / VOLUME

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the Content Area 'Measurement' an area which is allocated 15% of the total weight shared by the five content areas in Grade 4.
- This unit covers the meaning of the terms volume and capacity, measuring, estimating and comparing capacity, reading measurements and solving capacity problems.
- The purpose of this unit is to consolidate learners' sense of capacity, to understand the relationship between units of capacity and to read any measurements on a measure jug.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Measure in non-standard and informal units • Get a basic sense of how much litres and millilitres are • Start measuring with marked cups and spoons • Read measuring jugs on the lines that are numbered • Find capacity/volume by packing or filling containers • Compare volume and capacity with words "more" and "less"	• Use formal, standard measuring instruments eg measuring cups, -jugs and medicine teaspoons to measure volume • Get an exact idea of the units litre and millilitre and the ratio of these units (1:1000) • Start converting between units • Read measuring jugs on, and between the numbered calibration lines • Compare different volumes or capacities in numerical terms up to four digits • Understand the terms "volume" and "capacity" as well as the relationship between the two concepts	• Capacity and volume are not included in Term 3's schedule for Grade 5, however, the progression from Grade 4 is as follows: • Develop an understanding of why the volume of rectangular prisms is given by length multiplied by width multiplied by height • Learn the unit kilolitre in addition to litre and millilitre, as well as its relation to litre (1:1000) • Convert between units through formal calculations.

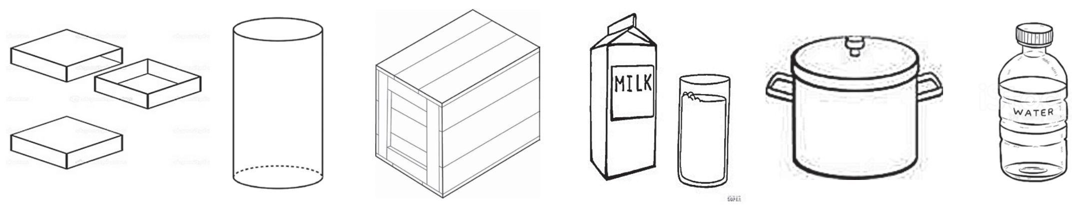
GLOSSARY OF TERMS

Term	Explanation / Diagram
Volume	The amount of space that an object takes up or occupies, which we measure in cubic units of length.
Capacity	The amount of substance that a container can hold in its inside, or the space that is inside a container, which we measure in units of capacity. For example, litres may be used if a liquid is inside the container.
Measuring instruments for capacity	Measuring spoons, measuring cups and measuring jugs are made and marked with the exact measure for the capacity of a container.
Unit of volume	The object or container is measured in units of length: metre, centimetre or millimetre for example.
Unit of capacity	The unit of measurement used for the capacity of a container. For example, millilitres or litres.

SUMMARY OF KEY CONCEPTS

The Two Concepts Capacity/Volume

1. Volume is the amount of space taken up by an object, and capacity is an object's maximum ability to hold a substance, like a solid, a liquid or a gas. Note that a container could have a capacity of 1 litre but may only be half full and would therefore hold 500ml.
2. We measure volume in cubic units of length (eg cm^3) and capacity in units like litres. The cubic measurement comes from the fact that 3 dimensions are used in finding the volume of a three-dimensional shape.
3. We can demonstrate volume and capacity with a water bottle of 500 ml. The amount of space that the bottle itself takes up, is its volume. Because the bottle is hollow on the inside, it has a capacity to hold another substance like 500 ml of water or two cups of sugar.



Measuring and Reading Capacity and Volume Using Marked Measuring Instruments

1. Measuring spoons, measuring cups and measuring jugs are made and marked with the exact measure for the capacity of a container.

A standard teaspoon has a capacity of 5 ml
A standard tablespoon has a capacity of 15 ml
A standard cup has a capacity of 250 ml

It is important to discuss with learners how to read a measurement. For example, if it is a jug, they should ensure that they are reading it with their eyes horizontal to the markings and not from above or below the markings.

2. Learners can spend time reading units of capacity from an item such as a measuring jug.

   Learners should be encouraged to look in their own kitchens at home for various measuring items and practice. This could include water into a jug or oil into a tablespoon for example.

   Below is an example of a recipe where many measurements are required.

   Discuss the recipe below (or one similar) with learners asking questions such as what measurement instrument would be used for the salt for example before telling them what would be used.

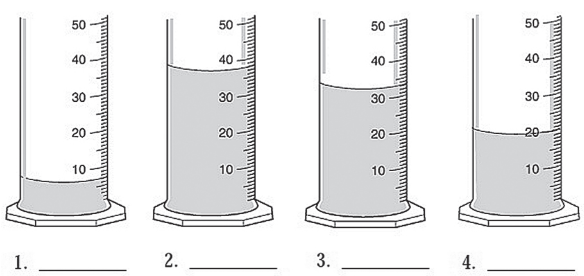
   Baking pancakes:

   - Measure out 375 ml water with a measuring cup into a bowl.
   - Break 2 eggs into the water.
   - Add 2.5 ml salt measured with a medicine spoon, to the eggs and water.
   - Measure 75 ml oil and pour that into the bowl as well.
   - Measure out 400 ml flour and pour that into the bowl.
   - Measure out 5 ml baking powder and add that to the rest of the ingredients.
   - Measure out 10 ml vinegar and add to the mixture.

Beat all the ingredients together until it is a smooth mixture that is almost as thick as cream and bake it in a hot, oiled pan. Make cinnamon sugar by mixing 5 ml cinnamon powder with 250 ml sugar. Sprinkle on pancakes.

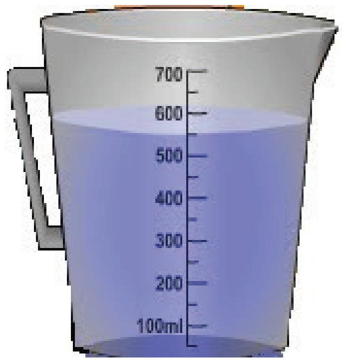
3. Learners read off the volume on numbered gradation lines and calculate what the unnumbered gradation lines and the spaces (intervals) in between the lines represent.

   Example:



4. Learners must be able to calculate unnumbered gradations up to 10 unnumbered.

   Example:



   The gradation lines on this jug are marked in every 100ml, but there is an extra line in between each of these. This divides each 'hundred' into two, therefore the extra marking shows an extra 50ml (100ml ÷ 2) each time.

   The liquid is filled to one space above the 500ml.

   The amount of liquid in this jug is therefore 500ml + 50ml = 550ml.

5. For example in the middle between 0 and 150ml is 75ml; at the quarters of 10 litres are 2.5 litres, 5 litres and 7.5 litres; and each gradation line between 0 and 500ml stands for 25ml, because there are 20, of which 250 is in the middle.

0_________________________|_________________________150ml

0___________|_____________|_____________|__________10litres

0__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|__|500ml

250ml

Syringes can also be used to demonstrate measurements to learners. The syringe below has a capacity of 100ml but if there was liquid in the end where the needle usually is, there would only be 20ml. If any learners have had an injection or perhaps even have a mother or father that is a nurse, the class may enjoy such a discussion and to share their own stories.

Understanding Capacities as Fractions of a Litre and as Decimals

1. Learners must be able to use ml to indicate various parts of a litre. They must know the following facts:
   - 1 litre = 1000 ml
   - $\frac{1}{2}$ litre = 500 ml
   - $\frac{1}{4}$ litre = 250 ml
   - $\frac{3}{4}$ litre = 750 ml
   - $\frac{1}{5}$ litre = 200 ml

2. Learners must be able to read and understand when fractions or decimals are used. The following list should be known well:
   - 0.25 = $\frac{1}{4}$
   - 0.75 = $\frac{3}{4}$
   - 0.5 = $\frac{1}{2}$
   - 0.2 = $\frac{1}{5}$

Calculating in the Context: Capacity

1. Converting between litres and millilitres

Examples:

1357 ml	= 1 litre and 357 ml
2 ¼ litres	= 2 250 ml

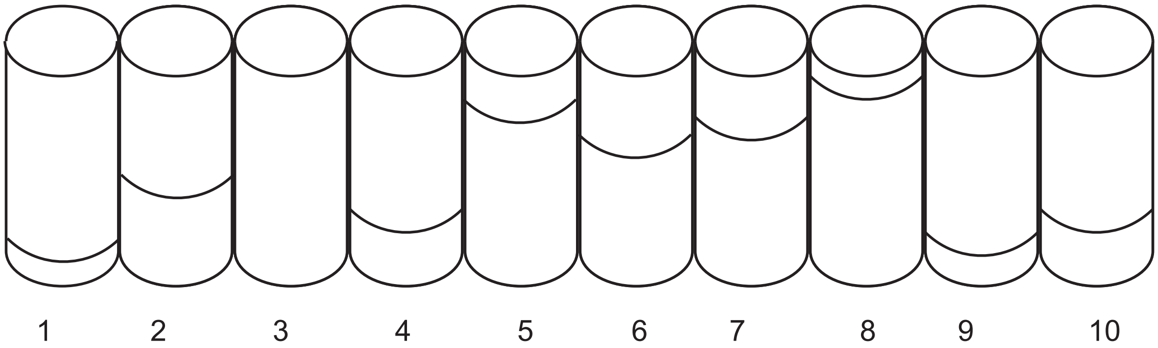
2. Estimating and rounding in the context of capacity

Examples:
Round off to the nearest litre: 8 642 ml (1 litre)
Round off to the nearest 100 litre: 3 579 litres (3 600 litres)

Learners should be able to visually estimate the amount of liquid in a container. In the example below they do this by comparing to other containers.

Examples:
All the containers below have a capacity of 1 litre. Container number 6 is half full.

- Which container has water the closest to ¾ litre? (5)
- Which container has water the closest to 400 ml? (2)
- Estimate how much water container number 1 has. (100ml)
- Estimate and compare the amount of water in containers number 4 and 10. (They both have about 250ml or 300ml)
- Estimate how much water container number 5 has. (750ml)
- Estimate how much water container number 7 has. (600ml)
- Which container has about 100 ml of water? (1 or 9)



TOPIC 2: COMMON FRACTIONS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Numbers, Operations and Relationships' an area which is allocated 50% of the total weight shared by the five content areas in Grade 4.
- For Term 3, this unit covers a range of problem solving contexts and diagrams with a focus on thirds, fifths and sixths.
- The purpose of this unit is to consolidate learners' understanding of equivalent forms and the magnitude of fractions as well as the use of fractions in contextual problems.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Use and name unitary and non-unitary fractions including halves, thirds, quarters, fifths, sixths and eighths • Recognise fractions in diagram form • Realise that two halves, three thirds, four quarters etc form one whole • Realise that two quarters and a half are the same, that two eighths and a quarter are the same • Write fractions as 1 half; 2 thirds; 4 fifths etc.	• Compare, order fractions with different denominators including halves, thirds, quarters, fifths, sixths, sevenths and eighths • Describe and compare common fractions in diagram form • Do addition calculations of fractions with the same denominators • Recognise and describe that division and fractions are equivalent concepts • Solve problems with fractions in context including grouping and equal sharing • Recognise and describe equivalent forms of fractions of which the denominator is a multiple of another	• Count forwards and backwards in fractions • Compare and order common fractions to at least twelfths • Do addition and subtraction calculations of fractions with the same denominators • Do fractions of whole numbers which result in whole numbers • Recognise, describe and use the equivalence of division and fractions • Solve problems with fractions including equal sharing and grouping • Recognise and describe equivalent forms of fractions of which the denominator is a multiple of another

GLOSSARY OF TERMS

Term	Explanation / Diagram
Common fraction	A fraction is a part or parts of a whole that has been shared equally into a number of parts. A fraction can also be a part of a number of things that have been divided into equal groups.
Denominator	The number of equal parts into which a whole has been divided, or the number of equal smaller groups into which a bigger group has been divided. We write that number under the fraction line. For example $\frac{2}{5}$.
Numerator	The number that tells us how many parts or groups we are dealing with. That number appears above the fraction line. For example $\frac{2}{5}$.
Fractions and whole numbers	When all the parts of something are together, they form a whole. $\frac{6}{6}$ is a whole, but $\frac{5}{6}$ is a fraction.
Fractions and mixed numbers	When we have a whole or a number of wholes and also a fraction, we call it a mixed number. For example $3\frac{1}{4}$.

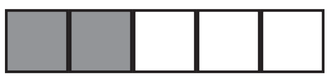
SUMMARY OF KEY CONCEPTS

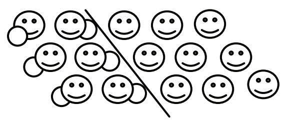
Introduction

We learn fractions up to sixths with diagrams, number symbols and verbal names of fractions.

1. Describing and Ordering Fractions
   a. A fraction is is a part of a whole which is shared equally into a number of parts.
   b. A fraction is also a part of a number of things that are divided into equal groups.

   Example:
   a. Two-fifths of the bar has been shaded.
   b. Two-fifths of the class play soccer





2. All the parts of something put back together, is a whole.
   Example:
   $\frac{6}{6}$ of the first pizza is the whole pizza, but $\frac{5}{6}$ of the second pizza is a fraction

3. When we have wholes and fractions, we have mixed numbers.
   Example:
   We have two whole pizzas ($\frac{6}{6}$ and $\frac{6}{6}$) and $\frac{5}{6}$ of a pizza ($\frac{17}{6}$).
   We write $2\frac{5}{6}$

4. The more parts something is divided up into, the smaller the parts become.
   Example:
   $\frac{1}{3} > \frac{1}{5}$ (compare fractions with numerator of one, but with different denominators)
   $\frac{2}{3} > \frac{2}{5}$ (compare fractions with the same numerator but different denominators)

$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$

$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

Teaching tip:
Learners understand the comparison if the teacher says the two fractions out loud: "Which one is bigger: one THIRD or one FIFTH?"

Adding Common Fractions With the Same Denominator

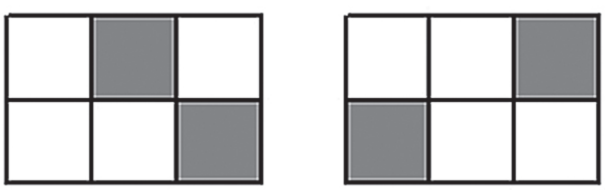
Add fractions with the same denominators using diagrams and also number symbols.

Example:

$$\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}$$

Example:

$$\frac{4}{6} + \frac{4}{6} = \frac{8}{6} = 1\frac{2}{6}$$



Equivalent Fractions

1. Explain equivalent fractions with diagrams and then
   write them down as an equation.
   Example: Fractions with denominators that can be divided by 3, all have exact thirds

Sixths: $\frac{2}{6} = \frac{1}{3}$ and $\frac{4}{6} = \frac{2}{3}$

Understanding Fractions as a Part of a Number of Items

1. Fractions are not only part of a whole, but there are fractions of numbers of items too.
   Example: Thirds of 6: I fold a paper strip in three. I call the parts that I have folded, thirds. My paper strip is 6 units long. One third of 6 is two.
   Two thirds of 6 is 4. Three thirds of 6 is 6!

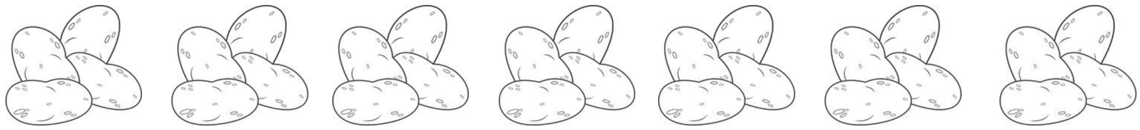
1	2	3	4	5	6
$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	

Example: Fifths of 15: I fold my paper strip in five. I call the parts that I have folded, fifths. My paper strip is 15 units long. One fifth of 15 is three. Two fifths of 15 is 6. Three fifths of 15 is 9, etc.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$		

Challenge problem with fractions

Mary gave three quarters of the potatoes to Grandmother. How many potatoes did her grandmother receive?



Solution: There are 28 potatoes. These can be divided into 4 (quarters) groups of 7. Three of these groups will be 3 x 7 which means Mary's grandmother will receive 21 potatoes.

TOPIC 3: ADDITION AND SUBTRACTION

INTRODUCTION

- This is a combined unit which runs for 1 + 5, i.e. 6 hours.
- It is part of the Content Area ‘Numbers, Operations and Relationships’ a topic which is allocated 50% of the total weight shared by the five content areas in Grade 4.
- This unit covers some number concepts and skills up to 4 digit numbers and arithmetic strategies for addition and subtraction.
- The purpose of this unit is to strengthen and expand learners’ existing number concepts and operations as a basis to master the more complex ideas and calculations following in Grade 5.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describe, order and compare whole numbers up to 999 • Order numbers from smallest to greatest and greatest to smallest • Add up to 999 and subtract from 999 downwards • For addition and subtraction use strategies of building up and breaking down; number lines; rounding off to tens	• Describe, order and compare whole numbers up to at least 5 digit numbers • Round off to 10, 100, 1 000 • Represent odd and even numbers to at least 1 000 • Add and subtract whole numbers of at least 4 digits • For addition and subtraction use strategies of building up and breaking down; number lines; rounding off and compensating; using addition and subtraction as inverse operations	• Describe, order and compare whole numbers up to at least 6 digit numbers • Round off to 5, 10, 100, 1 000 • Represent odd and even numbers to at least 10 000 • Add and subtract whole numbers of at least 5 digits • For addition and subtraction use strategies of building up and breaking down; number lines; rounding off and compensating; using addition and subtraction as inverse operations; adding and subtracting in columns

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole numbers	Whole numbers are numbers we use to count, including zero: 0,1,2,3,4….
Number line	On a number line, numbers are marked at equal intervals, starting at 0 on the left and going up in ones to the right as shown below: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
Ordering	To put numbers in their order of size or quantity. This could be in ascending order (smallest to biggest) or in descending order (biggest to smallest)
Comparing numbers	When comparing numbers, you may find one is bigger, smaller or the same as another; or you may find how much they differ.
Estimation	To find a value that is close enough to the right answer, usually with some thought or calculation involved. the calculation is most often mental. Rounding is a useful skill in estimating.
Digit	A digit is a symbol that we use to represent a quantity. There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 which we use in different positions in numbers.
Building up and breaking down	We can write all whole numbers >1 in terms of their parts. We expand a number in its parts, like 153 is 100 + 50 + 3.
Rounding off	Rounding means making a number simpler but keeping its value close to what it was. The result is less accurate, but easier to use. For example, 162 rounded to the nearest 10 is 160
Compensating	A calculation strategy for addition and subtraction where we change the second number to a "friendly number" like 30 and compensate for that later. Example: 426 + 28 --> 426 + 30 --> 456 – 2 = 454
Inverse Operations	Inverse operations are opposite operations that undo each other. for example 15 - 7 = 8 but 8 + 7 = 15

SUMMARY OF KEY CONCEPTS

Introduction

We do addition and subtraction both context free and in word sums using expanded notation.

Facts in addition and subtraction

1. When we multiply a number by zero, the result is zero.
   Example: 34 x 0 = 0

2. When we add numbers, it does not matter in which order we add them.
   Example: 15 + 16 is the same as 16 + 15. 15 + 16 = 16 + 15 = 31

3. When we add numbers, it does not matter how we group them to add them.
   Example:
   We can group 15 + 3 + 7 like this: either (15 + 3) + 7 or 15 + (3 + 7)

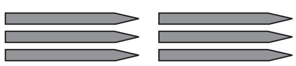
   This is a useful skill to add many numbers a little more quickly by finding pairs of numbers that would group to make an easier number to work with.
   For example, 12 + 3 + 11 + 17.
   17 + 3 = 20. Doing this calculation first makes the calculation simpler.

4. A number before the brackets, gets distributed to all numbers in the brackets:
   Example: 2 x (3 + 4) = 6 + 8

   Although it is possible to perform the calculation in the brackets first and then multiply by 2 an understanding that by distributing gives the same result (14 in this case) is an important concept for later in the senior phase when algebra is introduced.

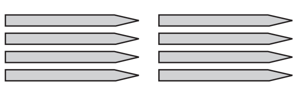
   The example above is demonstrated using a practical example representing the situation in diagram form:

   If two learners each had three red pencils and four yellow pencils,

   2 x 3 red pencils

   plus

   2 x 4 yellow pencils

   In total there are 14 pencils.

Inverse operations

We use addition to check if subtraction is correct, and subtraction to check if addition is correct

Subtraction is the inverse of addition; addition is the inverse of subtraction.

Example:
Add 89 to 567, the sum is 656 (567 + 89 = 656) and 656 – 89 = 567

Rounding

We increase or decrease a number to a 'friendly' approximate number. In other words a multiple of 10, 100 or 1000 and so on. We can round numbers to the nearest multiple of ten for example 38 ≈ 40 and 32 ≈ 30

Note the following calculation and how rounding can be used to assist learners to quickly estimate an answer:
43 + 21

A very rough estimate would be 40 + 20 which is 60. Here both numbers were rounded to the nearest 10.

A better estimate would be 43 + 20 which is 63. Here only one number was rounded to the nearest 10.

Estimating is an important skill for later when learners start using calculators. Learners should be encouraged to always estimate first so when an answer that doesn't seem correct is shown on the calculator they can check their work instead of just assuming that the answer must be correct because the calculator 'says so'.

Estimating

When we do a close guess of the actual answer, we estimate. We do some thinking and calculation in our heads but we do not actually calculate the answer. Rounding is a handy way of estimating.

Example:

There are three rows of bicycles at school. You count the first row, it is 9. That is almost 10 if you round it. There are three rows, so you estimate there are 30 bicycles. If you count the bicycles there are actually 9 + 8 + 9 = 26, which is close enough to your estimation of 30.

Topic 3 Addition and Subtraction

Addition and subtraction strategies

1. There are two ways in which we use expanded notation in addition:
   a. both parts are expanded
   b. only the second part is expanded

A. Expanded notation: break-down method (both parts expanded)

i. Horizontal: Expanded notation: break-down method (both parts expanded)

Example:
2 713 + 3 224

= 2 000 + 700 + 10 + 3 + 3 000 + 200 + 20 + 4	expand both numbers
= 2 000 + 3 000 + 700 + 200 + 10 + 20 + 3 + 4	group thousands, hundreds, tens, units
= 5 000 + 900 + 30 + 7	add each pair as grouped above
= 5 937	

ii. Vertical: Expanded notation: break-down method (both parts expanded)

Example:

2 713	+	3 224		
3	+	4	= 7	add units horizontally
10	+	20	= 30	add tens horizontally
700	+	200	= 900	add hundreds horizontally
2 000	+	3 000	= 5 000	add thousands horizontally
2 713	+	3 224	= 5 937	add totals vertically

B. Expanded notation: break-down method (only one part expanded)

Example:
2 713 + 3 224
2 713 + 3 000 ---> 5 713 + 200 ---> 5 913 + 20---> 5 933 + 4 ---> 5 937

It is important to note that there are NO equal signs between each of these calculations. Each part is NOT equal to the previous part.

2. There are two ways in which the expanded notation is used in subtraction:
   a. both parts are broken down or expanded
   b. only the second part (subtrahend) is expanded or broken down

A. Expanded notation (break-down method, both parts expanded)

i. Horizontal: Expanded notation (break-down method, both parts expanded)

Break down both parts, the first number in brackets (separated by + signs) followed by the subtrahend, (separated by – signs), then group together the Th's, H's, T's U's, each pair bracketed and separated by – signs, but brackets separated by + signs add the totals.

NB: START SUBTRACTING FROM THE UNITS

Example:

4232–1438

= (4 000+200+30+2)–1 000–400–30–8 First in brackets with + signs; second with – signs

= (4 000–1 000)+(200–400)+(30–30)+(2–8) Group and work from units backwards

= (4 000–1 000)+(200–400)+(20–30)+(12–8) Unit minuend borrows from tens minuend

= (4 000–1 000)+(100–400)+(120–30)+4 Tens minuend borrows from hundreds minuend

= (3 000 – 1 000) + (1 100 – 400) + 90 + 4 Hundreds borrows from thousands minuend

= (3 000 – 1 000) + 700 + 90 + 4

= 2 000 + 700 + 90 + 4

= 2 794

ii. Vertical: Expanded notation (break-down method, both parts expanded)

Break down both parts, one below the other in a column, the Th's, H's, T's U's of both numbers next to the other, subtract to the side, add the totals downwards. Leave lines open in between!

NB: START SUBTRACTING FROM THE UNITS

Example:

	4232	– 1438		
	2	– 8	= (cannot)	
	12	– 8	= 4	<---- Open line, filled in if/when needed
+ 20	~~30~~	– 30	= (cannot)	
	120	– 30	= 90	<---- Open line, filled in if/when needed
+100	~~200~~	– 400	= (cannot)	
	1 100	– 400	= 700	<---- Open line, filled in if/when needed
+3 000	~~4 000~~	– 1 000	= 2 000	
			=2 794	

B. Expanded notation (break-down method, only second number expanded)

NB: START SUBTRACTING FROM THE LARGEST, IE THE THOUSANDS

Example:

4 232 – 1 438
4 232 – 1 000 ---> 3 232 – 400 ---> 2832 – 30 ---> 2802 – 8 ---> 2 794

Compensating

We change the second number to a "friendly number" like 30 and adjust the answer. Compensation works best when the subtrahend is bigger than 5, especially with an 8 or a 9.

Example:

456 - 28 Because I know that 28 = 30 – 2:
456 - 28 → 456 - 30 → 426 + 2 = 428

(I have subtracted two more than I should have, so I add 2)

TOPIC 4: VIEWING OBJECTS

INTRODUCTION

- This unit runs for 2 hours.
- It is part of the Content Area 'Space and Shape (Geometry)' an area which is allocated 15% of the total weight shared by the five content areas in Grade 4.
- This unit covers viewing side- and top views and plan views of everyday objects.
- The purpose of this unit is to develop the mental ability to have only one surface area of a three-dimensional shape in view, yet complete the rest of the object imaginarily.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Experience and recognise that there are different views of the same everyday object • Identify everyday objects from different views • Read, interpret and draw informal maps or top views of a collection of objects • Find objects on maps	• Recognise and match different views of picture representations of the same everyday object • Identify everyday objects from different views • Interpret top views of scenes and plans	• Link the position of the viewer relative to the view of single everyday objects • Link the position of the viewer relative to the view of groups of everyday objects • Link the position of the viewer relative to the view of everyday scenes

GLOSSARY OF TERMS

Term	Explanation/diagram
View	What a person can see, or take into the eye, from where they are at that moment.
Position	When we view an object, there are two positions: The place from where a person looks, is the person's position. The place where the object is at which the person is looking, is the object's position.
Perspective	When we draw a 3D object on a 2D surface, we create perspective to make it look three dimensional.
Scale	Scale is used in plans of houses and for area maps, to draw something in a way that it is a small image of something large.

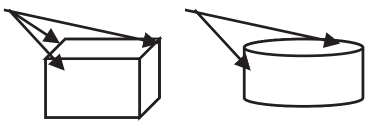
SUMMARY OF KEY CONCEPTS

Introduction

To view objects in Grade 4 includes
- creating a top view from a given side view of a real life situation
- viewing a large object like a house on a scaled diagram from different positions
- viewing a large area on a scaled map from a top view

Perspective

When we draw a 3D object on a 2D surface, we create perspective to make it look 3D and to give the right impression of its height, width, depth, and the



position where it is standing. The way we draw the shape, gives perspective like in the pictures above.

Scale

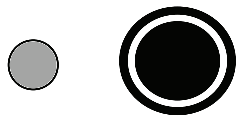
We use scale in plans of houses or for area maps, to draw something in a way that it is a small image of something large.
- In house plans, we take different views of the house and make drawings of those. The pictures must be on scale, for example one metre of the house is one centimetre on the plan. This is a scale of 1:100, because 1 metre is the same as 100 cm.

Topic 4 Viewing Objects

Viewing objects from different sides

1. Match views with pictures of objects and identify them from given points of view.
   Example:
   - If you are next to something, you look straight to it and you have a side view of it.

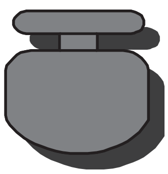
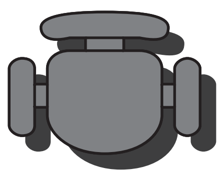
   - If you are above something, you look down on it and you have a top view of it.



Example:

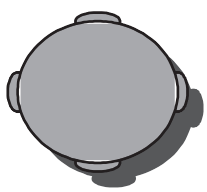
- Draw this car and its driver from a top view
- Draw this car and its driver from a front view
- Draw this car and its driver from a back view

2. Example:

3. a.  b. 

The two chairs in the pictures above, are viewed from the ______.
Draw these two chairs from the side.

Example:
This table is viewed from a ______ view.
Draw this table from the side.



Learners should practice these skills using ideas similar to those below.

Figure 1 demonstrates all the different views that are possible to draw form a 3-dimensional shape.

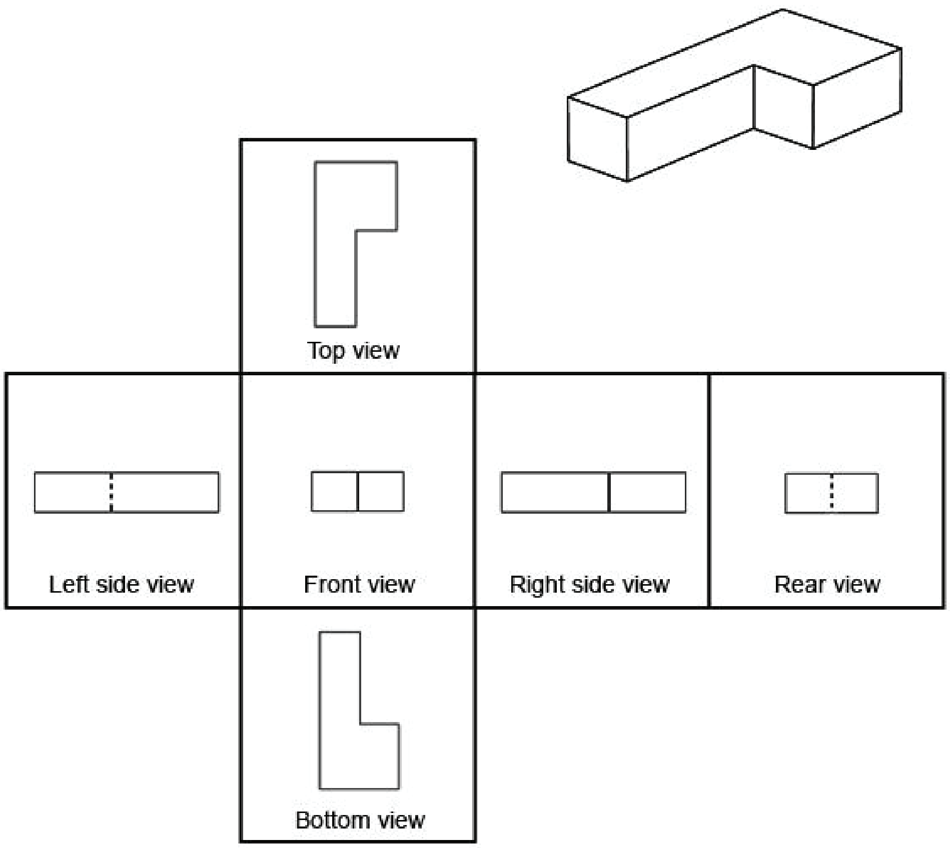


Figure 2 allows learners to choose a view that is correct and match it with headings such as front, right back and so on.

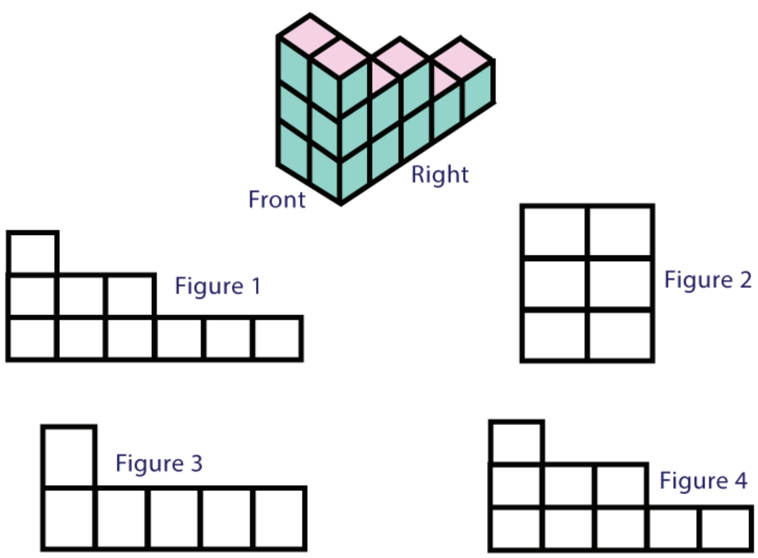
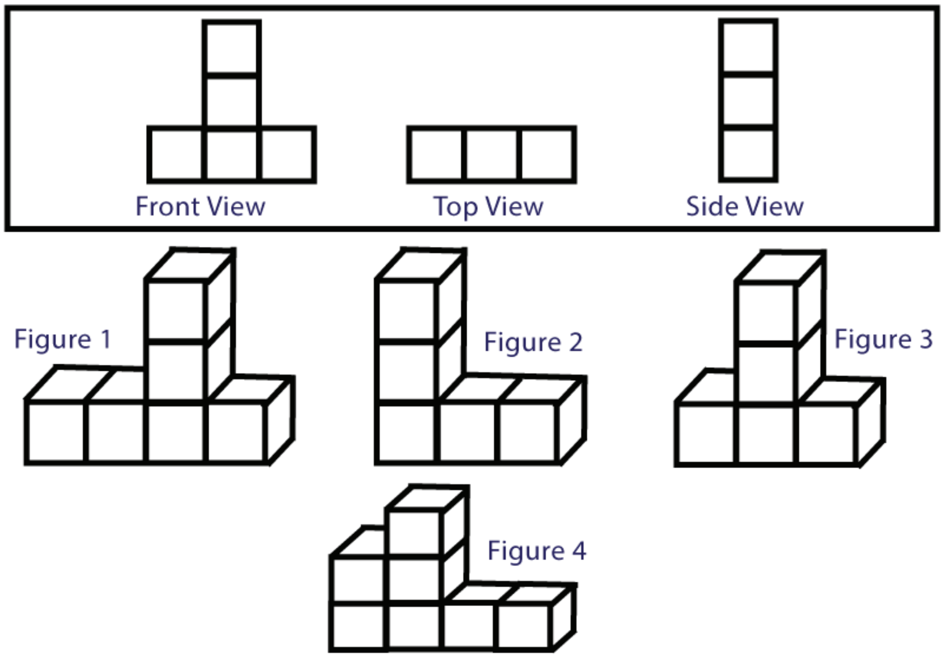


Figure 3 is similar to figure 2, but this time learners are given three views and must match them with their corresponding shapes.



Creating a top view from a given side view

Example:

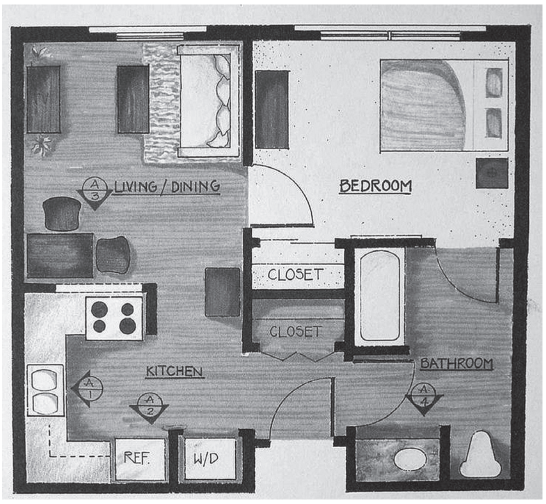
John sits here

Jane sits here

a. Draw a top view of this dinner table and show which is John's plate and Jane's plate.

b. To which side of John is the viewer standing in this picture? To which side of Jane is the viewer standing in this picture?

Understanding a house plan



Study the house plan
and answer the questions:

a. What furniture do you see in the bedroom?
b. How many doors does the bedroom have?
c. Is the closet inside the bathroom?
d. What do you see in the kitchen?
e. What kind of a view does this plan take?
f. Draw the house from the back.
g. Where do you think the television would stand? Explain why you think so.

TOPIC 5: PROPERTIES OF 2D SHAPES

INTRODUCTION

- This unit runs for 4 hours.
- It is part of the Content Area 'Space and Shape (Geometry)' an area which is allocated 15% of the total weight shared by the five content areas in Grade 4.
- This unit confirms the concepts learned previously, however learners' understanding is deepened to creating shapes themselves, not only defining given shapes.
- The purpose of this unit is to develop the ability to construct polygons by using straight lines and to observe the growing size of internal angles as the number of sides increase.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Range of 2D shapes includes circles, triangles, squares and rectangles • Recognise and name circles, triangles, squares and rectangles • Draw 2D shapes, circles, triangles, squares and rectangles • Describe, sort and compare 2D shapes in terms of straight and curved sides	• Range of 2D shapes includes circles, squares, and rectangles, regular and irregular polygons, triangles, pentagons and hexagons • Recognise, visualise and name 2D shapes in the environment and in geometric settings • Describe, sort and compare properties of 2D shapes in terms of straight and curved sides and the number of sides	• Range of 2D shapes includes circles, squares, rectangles, triangles, regular/irregular polygons, pentagons, hexagons and heptagons • Recognise, visualise and name 2D shapes in the environment and in geometric settings with a focus on regular and irregular 2D shapes • Describe, sort and compare properties of 2D shapes in terms of straight and curved sides and the number of sides, length of sides, angles in shapes, limited to right angles, acute angles and obtuse angles

GLOSSARY OF TERMS

Term	Explanation/diagram
Polygon	A plane figure with at least three straight sides and angles
Quadrilateral	A two-dimensional shape with four straight sides and four angles
Pentagon	A two-dimensional shape with five straight sides and five angles
Hexagon	A two-dimensional shape with six straight sides and six angles
Regular shapes	Regular shapes with straight sides are shapes of which all sides and angles are the same

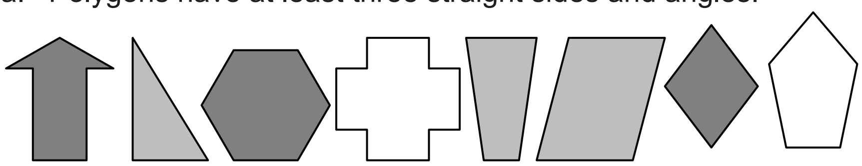
SUMMARY OF KEY CONCEPTS

Introduction

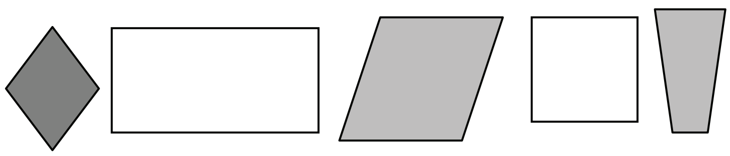
We focus on creating 2D shapes on grid paper, not only identifying existing given shapes.

Terminology

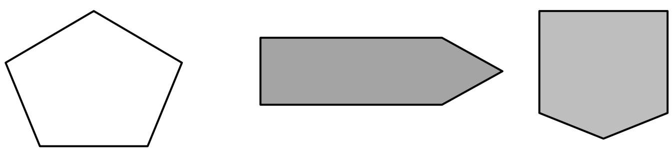
a. Polygons have at least three straight sides and angles.



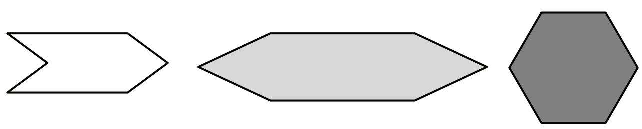
b. Quadrilaterals have four straight sides and four angles.



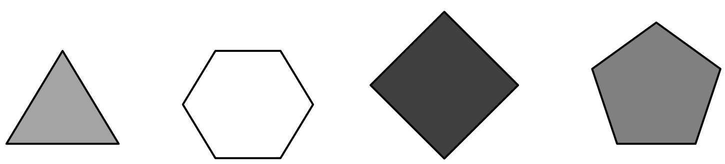
c. Pentagons have five straight sides and five angles.



d. Hexagons have six straight sides and six angles.



e. All sides of regular shapes are the same length and all angles the same size.



Creating 2D shapes

Learners should practice drawing shapes on grid paper and follow instructions similar to the examples below.

a. Square: Draw one square with sides of 7 units each

b. Rectangle: Draw two rectangles with a length of 9 units and a width of 4 units, facing in different directions

c. Right triangle: Draw a right triangle with one right side 8 units long and the second right side 6 units long

d. Rectangle built from two triangles: Build a rectagle from two triangles

e. Triangle; two sides equal: Draw a triangle with two sides of 10 units long each

f. Triangle with all sides equal: triangle with all sides 10 units long

g. Pentagon: Draw a pentagon of any size.

h. Hexagon: Draw a regular hexagon.

TOPIC 6: DATA HANDLING

INTRODUCTION

- This unit runs for 7 hours.
- It is part of the Content Area 'Data Handling' an area which is allocated 10% of the total weight shared by the five content areas in Grade 4.
- This unit covers the collection and organising of data, different forms of representing data including words, pictograms and bar graphs and analysing, interpreting and reporting data.
- The purpose of this unit is to develop learners' ability to see everyday events and situations in a diagrammatic- or graphical form with which numerical values can be associated.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Collect data through tally-marks • Represent data in bar graphs and pictograms (one-to-one correspondence) • Answer questions about the represented data	• Collect and record data through tally-marks • Represent data in words, bar graphs and pictograms (one-to-one correspondence) • Read and interpret the above representations as well as pie charts • Answer questions related to data categories • Critically read and interpret data • Summarise data verbally and in written form	• Arrange data from the smallest to the largest group • Represent data in words, pictograms (many-to-one correspondence), bar graphs, double bar graphs and pie charts • Answer questions related to data categories, -sources and contexts • Critically read and interpret data • Summarise data verbally and written, draw conclusions and make predictions • Determine the mode of the data set

GLOSSARY OF TERMS

Term	Explanation/diagram
Data	Facts about something that happens in this world (usually large numbers) and which we can count and give a number value to it
Data set	Consists of many pieces of information that are related to one another
Statistics	Statistics is created after one has worked through the data and you put it into tables, charts or graphs. Also, when you report the numbers or percentages, it is called statistics
Tally marks	A way of counting by only using ones. They are most useful in counting while things become more, like the score in a match or the cars passing
Pictograph	A graph/diagram where we use pictures to represent numbers in data
Bar graph	Numbered bars that show information in a graph
Pie chart	A Pie Chart is a special chart that uses "pie slices" to show relative sizes of data. The circle is divided into sectors, where each sector shows the relative size of each value.
Complete data cycle	A complete data cycle starts at a question. Then we collect, report, present and analyse the data to answer the original question

SUMMARY OF KEY CONCEPTS

Introduction

We work with one rich data set to apply the different concepts in the glossary of terms.

Collecting data

1. Tally is a do-word:
   Example: Sir Thabo coached the long distance runners at school from 2010 to 2015. Each year he tallied the number of runners in each grade who ran the big annual race.
2. Data collection: This means that Sir Thabo collected the data or the facts about the number of learners in each grade who did the 5 000 m race each year.
3. Tally table: The table that Sir Thabo used, looked like this:
   - A tally table has a heading that describes clearly exactly what data has been collected.
   - The table has columns going down and rows going to the sides.
   - We calculate the number of rows by adding one row for the column headings, the number of categories (in this case the four classes), and a row for the column total.
   - A tally table has three columns. The first column would be for the grades, the second column for the tallies and the third column for the number.

   Example: The first tally table is completed already. Now complete a table for 2013 in the same way by using the following information:

- 2010: Gr. 4: **2**; Gr. 5: **5**; Gr. 6: **8**; Gr. 7: **12**
- 2011: Gr. 4: **3**; Gr. 5: **7**; Gr. 6: **9**; Gr. 7: **13**
- 2012: Gr. 4: **4**; Gr. 5: **8**; Gr. 6: **11**; Gr. 7: **16**
- 2013: Gr. 4: **4**; Gr. 5: **8**; Gr. 6: **12**; Gr. 7: **16**
- 2014: Gr. 4: **5**; Gr. 5: **10**; Gr. 6: **15**; Gr. 7: **18**
- 2015: Gr. 4: **6**; Gr. 5: **10;** Gr. 6: **17**; Gr. 7: **21**

Runners of 5 000 m in 2010

Grade	Tally	Number
Grade 4	||	2
Grade 5	𝍸	5
Grade 6	𝍸 |||	8
Grade 7	𝍸 𝍸 ||	12
Total		**27**

Runners of 5 000 m in 2013

Grade	Tally	Total
Grade 4		
Grade 5		
Grade 6		
Grade 7		
Total		

Representing data

We can present data in different graphic forms:

1. Pictograph or pictogram:

- A pictogram has a heading in the first row, saying exactly what data is presented.
- It has two columns with as many rows as the categories of data.
- The first column has the names of the categories.
- The second column has the pictures used to represent a certain number of people.
- Note that a key is required so that information can be read from the pictograph accurately (one stick-man does not always represent 1 learner. If there were many learners involved, one stick-man could represent 5 learners for example).

Number of Grade 4 Learners that Ran 5 000 m from 2010-2015	
2010	🧍 🧍
2011	🧍 🧍 🧍
2012	🧍 🧍 🧍 🧍
2013	🧍 🧍 🧍 🧍
2014	🧍 🧍 🧍 🧍 🧍
2015	🧍 🧍 🧍 🧍 🧍 🧍
Key: 🧍 = 1 learner	

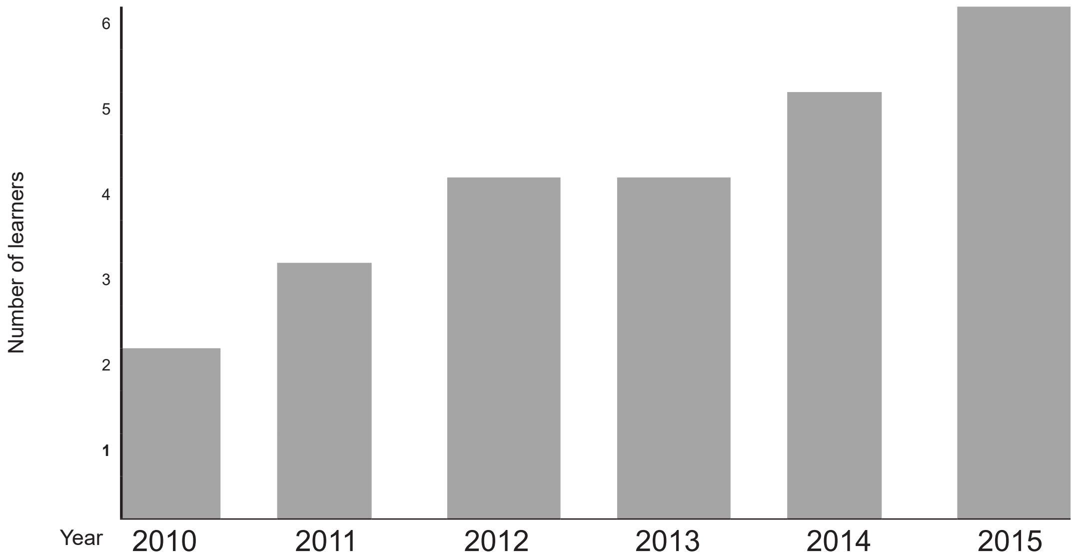
Now complete the pictogram or pictograph for Grade 6 too.

Number of Grade 6 Learners that Ran 5 000 m from 2010-2015	
2010	
2011	
2012	
2013	
2014	
2015	

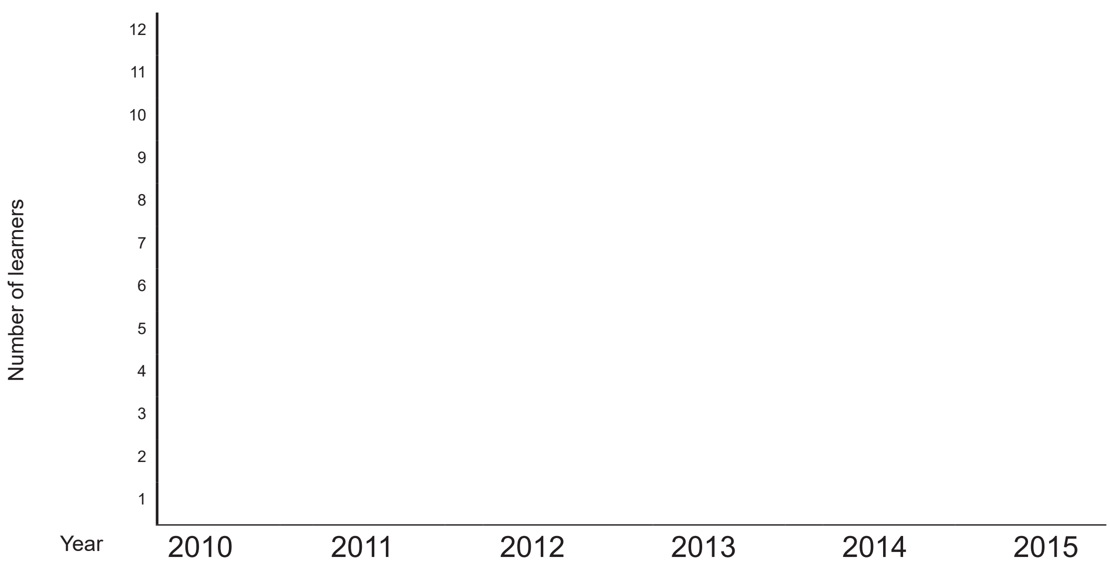
2. Bar graph:

- A bar graph has a **heading** to describe the data that is presented in the bar.
- A bar graph has a **horizontal line** and a **vertical line** that meet each other at **0**.
- The horizontal axis (x-axis) has the **names** of the categories of data (like the year or the grade).
- The vertical axis (y-axis) has the **number** value that goes with the categories.
- In a bar graph the **bars do not touch** each other and have a **space** between bars.

a. Number of Grade 4 learners who ran 5000m per year from 2010 - 2015



b. Number of Grade 5 learners who ran 5000m per year from 2010 - 2015

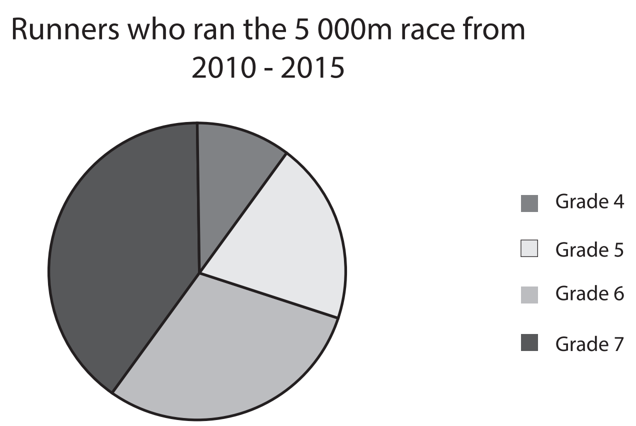


3. Pie chart:
   - Data can be represented in the form of a circle that is cut into sectors (slices of the 'pie')
   - There should always be a heading which shows clearly what data is being represented
   - Each sector should be clearly labelled OR a key on the side should show what each sector represents

   Example:

   The total number of learners who ran the race, was 240.

   - 24 of the 240 learners were Grade 4s, that is $\frac{1}{10}$.
   - 48 of the 240 learners were Grade 5s, that is $\frac{2}{10}$.
   - 72 of the 240 learners were Grade 6s, that is $\frac{3}{10}$.
   - 96 of the 240 learners were Grade 7s, that is $\frac{4}{10}$.



Learners need to be able to answer questions from a given pie chart. For example, from the above chart learners should be able to say which grade had the most or least learners or be able to see that the Grade 4 and Grade 7 learners make up half of the total learners and so on.

Once learners have covered and practiced all the basic skills, they should do a project where they work through the entire data cycle.

They can work in pairs to develop a question, collect data then organise it before representing the data in a suitable format and describing their findings.

There is a good resource for Grades 4 to 7 at this address:
http://academic.sun.ac.za/mathed/malati/3primdat.pdf

TOPIC 7: NUMERIC PATTERNS

INTRODUCTION

- This unit runs for 4 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' an area which is allocated 10% of the total weight shared by the five content areas in Grade 4.
- This unit covers the relationships between input and output values, the rules of patterns and equivalent forms of representing patterns.
- The purpose of this unit is to develop learners' sense of a number pattern of which the input value is the independent variable, the output value is the dependent variable and the relationship between the two values is determined by the rule for the pattern.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Complete, extend and describe simple number sequences in words • Create and describe own patterns	• Investigate and extend numeric patterns looking for relationships and rules • Find a constant difference in a numeric pattern • Find a constant ratio in a numeric pattern • Describe the relationship in learner's own words • Create and describe own patterns • Complete flow diagrams with two actions/double rule • Understand that the input value is derived from the inverse operations than those of the rule	• Investigate and extend numeric patterns looking for relationships and rules • Find a constant difference in a numeric pattern • Find a constant ratio in a numeric pattern • Describe the relationship in learner's own words • Create and describe own patterns • Complete flow diagrams with two actions or a double rule • Understand that input value is derived from the inverse operations of the rule's

GLOSSARY OF TERMS

Term	Explanation/diagram
Number pattern	A number pattern is a list of numbers that follow a certain rule.
Sequence	A sequence is an ordered list of diagrams or numbers which form a pattern. We can call a number pattern a sequence.
Term	Term is the position of a number in the sequence, for example, in 2, 5, 8, 11, 14, 17, the number 8 is in the third position from the first number, and it is therefore term 3.
Input value	In a number pattern where the numbers relate to each other, you do something to the number you choose (the input value) to produce the second number (the output value).
Rule	The rule of the pattern is a description of what calculations we do with the input number to find the output number.
Output value	The output value is the answer that we calculate when we apply the rule to the input value.
Flow diagram	A flow diagram is a visual way to write a number pattern, with the input values to the left, the rule in the middle and the output values to the right.
Inverse operations	Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

SUMMARY OF KEY CONCEPTS

Introduction

There are two ways in which we can look at pattern in a sequence:

1. How does a number relate to the next number in the sequence?

   Example: 2, 5, 8, 11, 14… (each following number is three more than the previous number). Each number in a sequence is called a term. The term is the position of the number in the sequence: 8 is in the third position, and it is therefore term 3.

2. How does a number change to form the next number? We do the same operation for each number to form the next number, for example multiply by 2 and subtract 1 (x 3 - 1).

Number patterns

A number pattern is a list of numbers or a sequence of numbers that follow a certain pattern, for every following number. We can call a number pattern a sequence.

Example: 2, 5, 8, 11, 14, 17, ... is a sequence. It starts at 2 and increases by 3 every time.

Input value

The input number can be any number: the counting numbers, any whole number, a fraction, or zero, like $\frac{1}{2}$; 5; 0;... or any of millions of options.

The rule of the pattern

What we do to our input number (adding, subtracting, multiplying or dividing) each time, we call the rule of the number pattern.

Output value

For all the numbers that we choose, we do the same to get the output value.

Example:

Input value	Rule	Output value
2	x 3 – 1	5
8	x 3 – 1	23

Number Patterns with Terms that are Related

These number patterns are related like this: 4 → 7 → 10 → 14...
(sideways to the next one)

Teaching tip:
The first four questions to ask in understanding a number pattern, are:

- "Do I add the same number each time?" (Ascending: constant difference)
- "Do I subtract the same number each time?" (Descending: constant difference)
- "Do I multiply by the same whole number each time?" (Ascending: constant ratio)
- "Do I divide by the same whole number each time?" (Descending: constant ratio) (however, it is important to note that when dividing by a number it is the same as multiplying by its inverse. For example, divide 2 is the same as multiplying by $\frac{1}{2}$)

1. Number patterns with a constant difference

   a. Ascending patterns formed through addition:
      For example: 4; 7; 10;... "The pattern starts at 4 and is increased by 3 (+3) each time"

   b. Descending patterns formed through subtraction:
      For example: 84; 81; 78;... "The pattern starts at 84 and is decreased by 3 (- 3) each time."

2. Number patterns with a constant ratio

   a. Ascending patterns formed through multiplication:
      For example: 3; 6; 9;... "The pattern starts at 3 and becomes 3 more each time"
      The rule of the pattern is: Each term is multiplied by three (x 3)

   b. Descending patterns formed through division:
      For example: 144; 72; 36;... "The pattern starts at 144 and becomes 2 times less each time" (÷ 2)

3. Number patterns with neither a constant difference, nor a constant ratio
   For example: 4; 5; 8; 13; 20; ...
   "The pattern starts at 4 and becomes 1 more the first time, 3 more the second time, 5 more the third time. Each time the next odd number is added."
   For example: 2; 3; 5; 7; 11; 13; 17;... "All the numbers in the pattern are prime numbers."

Number Patterns with Input and Output Numbers

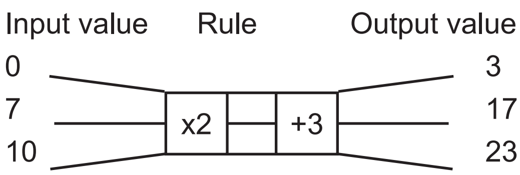
1. We choose input numbers and do the same thing (a rule) with all the numbers we choose to get output numbers as answers.

   For example: The rule is: multiply by 2 and add three (x 2 + 3).

Input number	0	7	10
	↓	↓	↓
Output number	3	17	23

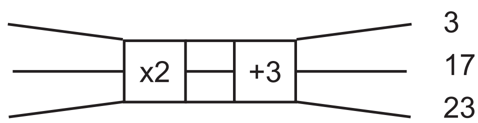
   We can apply the rule to any input number. The arrow shows the rule that we apply.

2. In a flow diagram it would look like this:



Inverse operations

If we have the output value and the rule of a number pattern, we can get the input number by doing the inverse operations of the rule (from right to left in the flow diagram).



3 – 3 → 0 ÷ 2 → 0
17 – 3 → 14 ÷ 2 → 7
23 – 3 → 20 ÷ 2 → 10

TOPIC 8: MULTIPLICATION

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Numbers, Operations and Relationships' an area which is allocated 50% of the total weight shared by the five content areas in Grade 4.
- This unit covers various strategies to multiply at least two digit numbers by two digit numbers.
- The purpose of this unit is revision of the work done in Term 2 and to recognise the commutative, associative and distributive properties of number.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Multiply up to 3 digit- by one digit numbers • Multiply through multiple addition, estimating, doubling and halving, building up and breaking down • Round off and estimate up to 999	• Multiply at least 2 digit- by two digit numbers • Multiply using estimation, doubling and halving, building up and breaking down, rounding off and compensating • Round off and estimate up to at least 9 999	• Multiply at least 3 digit- by two digit numbers • Multiply using estimation, doubling and halving, building up and breaking down, rounding off and compensating • Round off and estimate up to at least 99 999

GLOSSARY OF TERMS

Term	Explanation / Diagram
Multiplication	A short way of adding more than one of the same number together. Example: 4 + 4 + 4 + 4 + 4 + 4 + 4 = 28 or 7 x 4 = 28
Multiples	A number made up by multiplying two other numbers.
Factors	A factor is a whole number that will divide exactly into another number without a remainder.. The factors of a number are those numbers that were multiplied to make that number.
Halving	To divide a number into two equal parts, which is the same as dividing the number by two.
Doubling	To multiply a number by two, or to add the same number to it, so that the answer is twice as many as the number.
Rounding off Symbol	When one number is not exactly equal to, or the same as another number, we use the symbol ≈ to indicate that it is approximately, or almost the same as the other when we round off or estimate.

SUMMARY OF KEY CONCEPTS

Multiplication of at least a 2 digit number by a 2 digit number.

1. We can multiply by repeated addition, but it is only useful for small multipliers. It becomes too long when the multiplier is a 2 digit number. This method can work in the case below because the multiplier is only a one digit number.

   Example: 246 x 4

   = (200+200+200+200)+(40+40+40+40)+(6+6+6+6)
   = 800 + 160 + 24
   = 984

2. There are three ways to break down a number to multiply: the multiplier is either the sum, the difference or the product of other numbers.

   Example:
   We want to multiply 31 by 18:

   a. 18 is the sum of two numbers: 18 = 10 + 8
   b. 18 is the difference between two numbers: 18 = 20 – 2
   c. 18 is the product of two numbers: 18 = 6 x 3

a. 18 is the sum of two numbers: 18 = 10 + 8	b. 18 is the difference between two numbers: 18 = 20 – 2	18 is the product of two numbers: 18 = 6 x 3
31 x 18 = 31 x (10 + 8) = (31 x 10) + (31 x 8) = 310 + 248 = 558	31 x 18 = 31 x (20 – 2) = (31 x 20) – (31 x 2) = 620 – 62 = 558	31 x 18 = 31 x 6 x 3 = 31 x 6 x 3 = 186 x 3 = 558

Note that in the first two examples, learners need to have a good understanding of the distributive law.

3. We can double and halve in some cases to multiply, but that works well only in cases where one of the numbers is a multiple of 2, 4, 8 or 16.

Example: 96 x 35

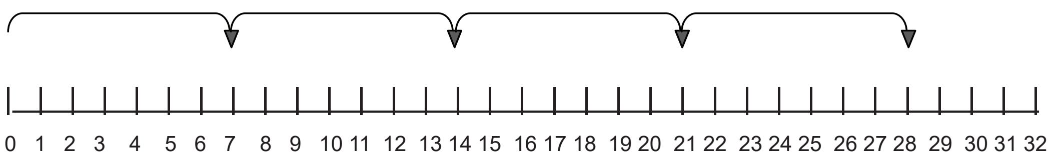
Halving	Doubling
96	35
48	70
24	140
12	280
6	560
3	1120
3 x 1120 = 3 360	

Rounding off symbol

When we round off or estimate, one number is not exactly equal to, or the same as another number so we cannot use the = symbol. Then we use the symbol ≈ which looks a little like =, but is not the same. This indicates that it is approximately, or almost the same as the other number.

For example:
1234 rounded to the nearest 100
1234 ≈ 1230

Multiples and multiplication by 0
Teaching tip: Explain this multiplication fact with a number line:



7 is the first multiple of 7; 14 is the second multiple of 7; 21 is the third multiple of 7; 28 is the fourth multiple of 7

7 x 1 = 7 7 x 2 = 14 7 x 3 = 21 7 x 4 = 28

Multiplication by 1

One multiplied by, or divided into a number does not change that number: one is the identity element for multiplication and division.

Example: 

Four spiders each has one head.

So four spiders have four heads altogether 4 x 1 = 4

Example: 

Four spiders are sharing one web.

The web has four spiders staying in it now, because 4 ÷ 1 = 4

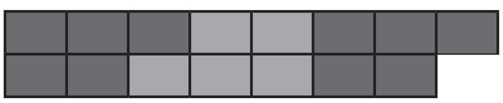
A multiplication investigation of odd and even numbers

1. Even x even = ________

   Image of 4 x 8

2. Odd x odd= ________

   Image of 3 x 5



3. Even x odd= ________

   Image of 4 x 7

Learners should be able to see when the answer is even or odd by first noticing the visual blocks that make an even or an odd number. Thereafter, it should be clear to learners that these rules must always be true.

TOPIC 9: NUMBER SENTENCES

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' an area which is allocated 10% of the total weight shared by the five content areas in Grade 4.
- This unit covers the transformation of a mathematical problem that was stated verbally, to a mathematical statement containing all the elements of the problem and solving the problem.
- The purpose of this unit is to introduce the concept of algebraic expressions and equations.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• This topic was not covered in Grade 3	• Introduction to algebraic expressions • Write number sentences to describe a problem situation • Solve and complete number sentences by inspection and trial and improvement • Check solution by substitution	• Write number sentences to describe a problem situation • Solve and complete number sentences by inspection and trial and improvement • Check solution by substitution

GLOSSARY OF TERMS

Term	Explanation / Diagram
Mathematical problem	A mathematical problem is a real world problem or a problem that we can solve with numbers and with the methods of mathematics.
Solving by inspection	A method of solving a number sentence by looking at it carefully and thinking logically what the solution can be without showing calculations.
Trial and improvement	A method of solving a number sentence by trying out several methods or possible solutions until you are satisfied with the answer.
Algebraic expression or number sentence without an = sign	An algebraic expression is a number sentence that can contain ordinary numbers, an unknown which we write as $\square$ and signs like +, - , x and ÷, for example $\square$ + 4 (We do not know what the value in $\square$ is)
Algebraic equation or number sentence with = sign	An algebraic equation is a number sentence with an equal sign, but where one or more of the elements are unknown like: $\square$ + 4 = 7
Substitution	Substitution is a method to solve a problem or to check if your solution is correct. In this method, solve an equation with one unknown, then substitute that solution in the equation to see if it works.

SUMMARY OF KEY CONCEPTS

A mathematical problem:

Example:
Thato read 46 pages in 2 hours. How many pages did he read in an hour?
Solution: 46 ÷ 2 = □

Solving by inspection:

Example:
When we look at this equation: 15 + □ = 19, we can "see" that the solution is 4 without focusing on the actual calculation.

Trial and improvement:

Example:
In the number sentence 234 – □ = 56
Try to subtract 200, then the answer is 34;
We see that the answer is about 20 too low;
We can improve the answer by subtracting 20 less, that is 180;
That gives us 54, which is two too few;
Therefore we subtract two less, which is 178.

An algebraic expression:

Example:
□ + 4 (We do not know what the value in □ is)

An algebraic equation:

Example:
□ + 4 = 7 (Now we can find the value in □)

Substitution:

Example:
□ + 4 = 7
Solution: □ = 3
Substitute 3 in □: 3 + 4 = 7

Topic 9 Number Sentences

Problem Types in Grade 4

1. Addition: (Number + number = sum)

a. Unknown number + known number = known sum

Example:Jim had some money and he received R25 more, now he has R68. How much money did Jim have in the beginning?

Number sentence: □ + R25 = R68
OR R25 + □ = R68
OR R68 – R25 = □

b. Known number + unknown number = known sum

Example: Jim had R43 and he received some more money, now he has R68. How much money did Jim receive?

Number sentence: R43 + □ = R68
OR □ + R43 = R68
OR R68 – R43 = □

c. Known number + known number = unknown sum

Example: Jim had R43 and he received R25 more. How much money does Jim have now?

Number sentence: R25 + R43 = □

2. Subtraction: Number – number = difference

a. Unknown number – known number = known difference

Example: Thabo's father had a number of sheep and after he sold 37, he was left with 56. How many sheep did Thabo's father have?

Number sentence: □ – 37 = 56
OR 37 + 56 = □

b. Known number – unknown number = known difference

Example: Thabo's father had 93 sheep. After he sold some of them, he was left with 56. How many sheep did Thabo's father sell?

Number sentence: 93 – □ = 56
OR □ + R56 = R93
OR R56 + □ = R93
OR R93 – R56 = □

c. Known number – unknown number = known difference

Example: Thabo's father had 93 sheep and he sold 37. How many sheep did Thabo's father have left?

Number sentence: 93 – 37 = □
OR □ + R37 = R93
OR R37 + □ = R93

It is important that learners notice how sometimes an addition question needed subtraction skills to find the answer and that sometimes a subtraction question needed addition skills to find the answer. Inverse operations are the key to solving many problems.

3. **Multiplication:** Number x number = product

a. Unknown number x known number = known product

Example:School B has 216 learners, which is 6 times as many as School A. How many learners does School A have?

Number sentence: □ x 6 = 216

b. Known number x unknown number = known product

Example: School A has 36 learners and School B has 216. How many times more learners does School B have than School A?

Number sentence: 36 x □ = 216
OR □ x 36 = 216
OR 216 ÷ □ = 36

Known number x known number = unknown product

Example: School A has 36 learners and School B has 6 times more learners. How many learners does school B have?

Number sentence: 36 x 6 = □
OR □ ÷ 36 = 6
OR □ ÷ 6 = 36
OR 6 x 36 = □

4. **Division:** Number ÷ number = quotient

a. Unknown number ÷ known number = known quotient

Example: Susan makes packets of 7 apples from a box of apples. She makes up 13 packets. How many apples were in the box?

Number sentence: □ ÷ 7 = 13
OR □ ÷ 13 = 7
OR 7 x 13 = □

b. Known number ÷ unknown number = known quotient

Example: Susan makes up exactly 13 packets from a box of 91 apples. How many apples are in each packet?

Number sentence: 91 ÷ □ = 13
OR 91 ÷ 13 = □
OR 13 x □ = 91

c. Known number ÷ known number = unknown quotient

Example: From a box with 91 apples, Susan makes up packets of 7 apples each. How many packets of apples does she make up?

Number sentence: 91 ÷ 7 = □
OR 91 ÷ □ = 7
OR 7 x □ = 91

It is important that learners notice how sometimes a multiplication question needed division skills to find the answer and that sometimes a division question needed multiplication skills to find the answer. Inverse operations are the key to solving many problems.

Setting up a number sentence

1. Setting up number sentences from word problems: We use the above problem structures in Grade 4 for learners to set up the number sentence from a given problem.

   Learners must also be able to set up number sentences from context free number problems.

   Example: There is a number that is 9 more than 23. What is that number?
   $23 + 9 = \square$ OR $\square - 9 = 23$ OR $\square - 23 = 9$

   Example: A number is 15 less than 38. What is that number?
   $38 - 15 = \square$ OR $\square + 15 = 38$

   Example: There is a number that is 7 times more than 22. What is that number?
   $22 \times 7 = \square$ OR $\square \div 7 = 22$

   Example: If I divide a certain number by 4, the answer is 43. What is that number?
   $\square \div 4 = 43$ OR $43 \times 4 = \square$

Solving number sentences

After setting up the number sentence, learners can solve the problem, or they can solve a number sentence that has already been set up.

Example: $4 \times \square + 15 = 59$

To do this successfully, learners will need to recognise that inverse operations will be required.

For the above example, learners may ask, what number subtract 15 gives me 59? Or some may notice that this is the same as 59 subtract 15, then the new calculation required would be $4 \times \square = 44$

Now learners would ask, what multiplies by 4 to get 44? Or some may even realise that they could also ask, 44 divided by what gives me 4?

These skills are essential to being confident in mathematics through to the senior phase.

Substituting the solution into the number sentence

After solving the problem, learners can replace the solution into the unknown space of the number sentence to check for correctness of the solution.

For example, if 11 is the answer that learners decided was correct, they could now write

4 x 11 + 15. They should do this calculation and if the solution is 59 then they will know that they have the correct answer.

If not, they need to be encouraged to try again as most learning is done through making mistakes.

TOPIC 10: TRANSFORMATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area 'Space and Shape' an area which is allocated 15% of the total weight shared by the five content areas in Grade 4.
- This unit covers the creation of composite 2D shapes, tessellations, rotation, reflection and translation as well as describing patterns in the environment.
- The purpose of this unit is to introduce learners to various forms of transformations and the effect that it has on 2D shapes.

SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise and experience lines of symmetry by folding geometric and non-geometric 2D shapes • Observe and recognise symmetry and transformations in nature and in the environment	• Recognise, draw and describe lines of symmetry in 2D shapes • Create composite 2D shapes by putting together various 2D shapes with line symmetry • Tessellate patterns with 2D shapes, some with line symmetry • Describe patterns in terms of the line of symmetry with an informal idea of the transformations, reflection, translation and rotation • Observe and recognise symmetry and transformations in nature and the environment	• Recognise, draw and describe lines of symmetry in 2D shapes • Use transformations to build composite 2D shapes by tracing and moving 2D shapes by rotation, by translation or by reflection • Use transformations to tessellate patterns with 2D shapes • Observe and recognise symmetry and transformations in nature and in the environment

GLOSSARY OF TERMS

Term	Explanation / Diagram
Symmetry	The quality of having two parts that match each other.
Tessellation	A pattern made of one or more shapes: • the shapes must fit together without any gaps • the shapes should not overlap
Transformation	A change in a 2D shape, where its appearance, position or orientation (direction) changes. Different types of transformation are reflection, translation and rotation.
Reflection	A transformation in which a geometric figure is reflected across a line, creating a mirror image.
Translation	A type of transformation where the original image is repeated, but has moved its position to the left or the right, and/or up or down.
Rotation	The original image is turned around about a point, clockwise or anticlockwise.
Composite Shapes	Shapes that are made up from a number of other shapes.
Tangram	A Chinese geometrical puzzle consisting of a square cut into seven pieces which we can arrange to make various other shapes.

SUMMARY OF KEY CONCEPTS

Symmetry

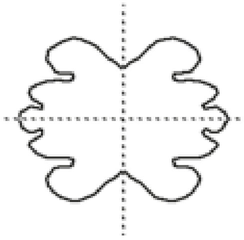
1. Symmetry is a quality of some 2D shapes. This means they will have at least two parts that will be exactly the same size and shape.

   The line of symmetry is the line that cuts the shape into these equal parts. A shape could have one line of symmetry, no lines of symmetry (it isn't a symmetrical shape) or more than one line of symmetry.

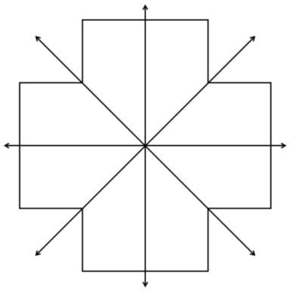
   Example – no line of symmetry

   Example – one line of symmetry

   Example – two lines of symmetry



   Example – four lines of symmetry

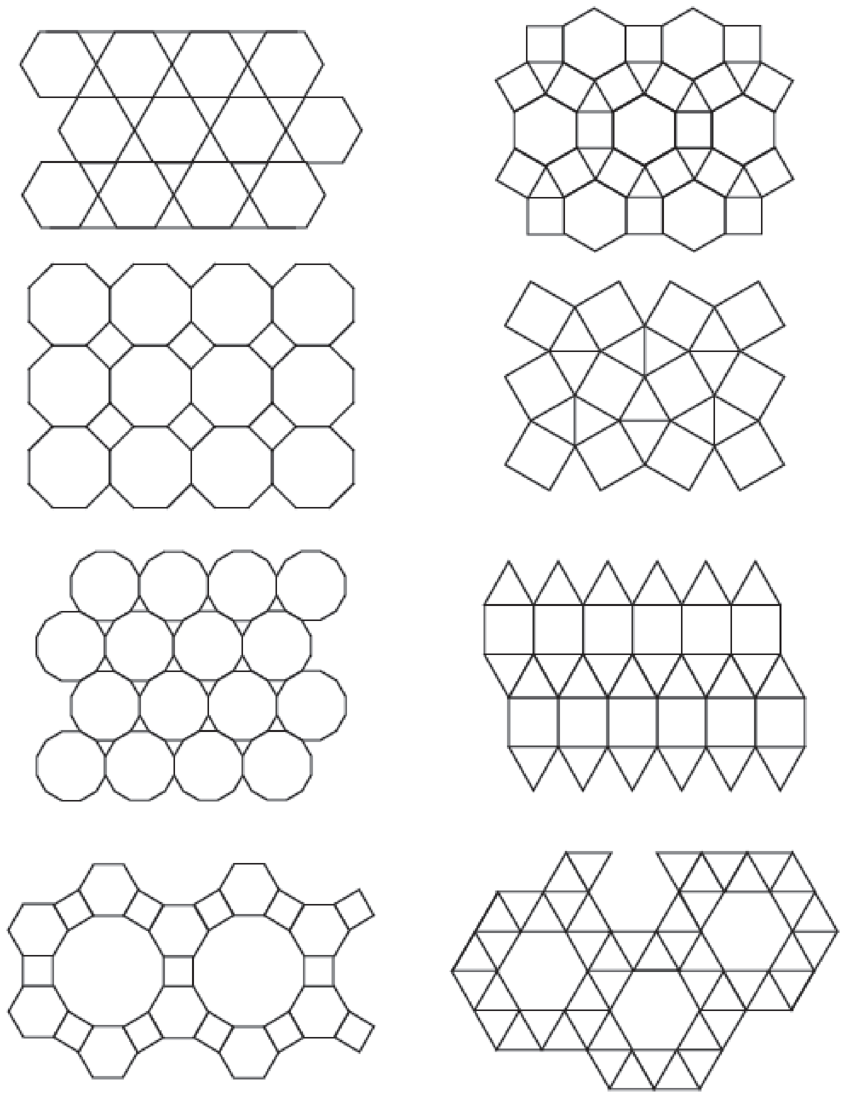


   Ask learners to discuss with a partner how many lines of symmetry they think a circle might have.

Tessellation

Tessellation is an arrangement of 2D shapes, especially of polygons, closely fitted together in a repeated pattern without gaps or overlapping.

Examples:

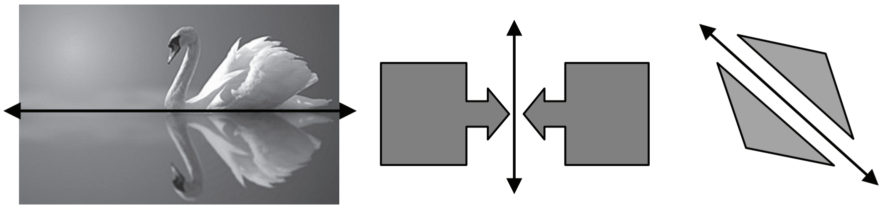


Teaching tip: Tesselations is an excellent choice of topic to link with Art. Learners enjoy creating and then colouring their own tesselations.

Transformations

Reflection is a type of transformation where the original image is repeated, as if in a mirror. We can reflect the image along a horizontal line, a vertical line, or a diagonal line.

Examples:

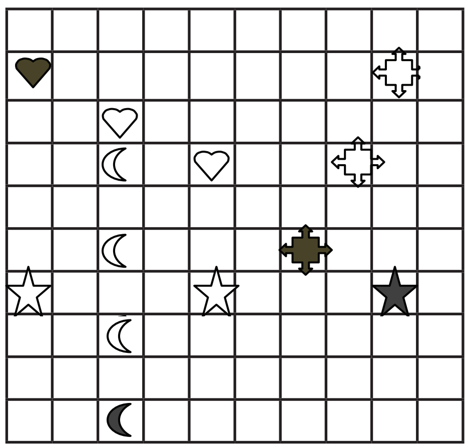


Along a horizontal line   Along a vertical line   Along a diagonal line

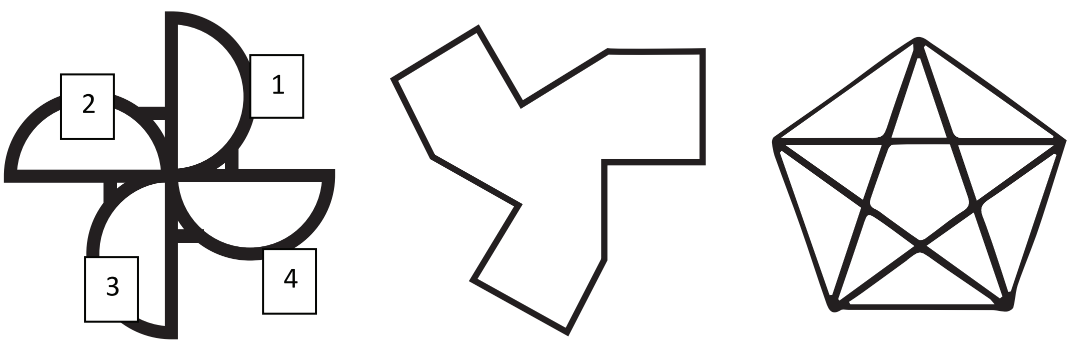
2. Translation is a type of transformation where the original image is repeated, but has moved its position to the left or the right, and/or up or down.

   Examples:

   - The heart has moved two right and one down each time;
   - The shape with arrows has have moved two up and one right each time;
   - The moon has moved two up each time;
   - The star has moved four left each time

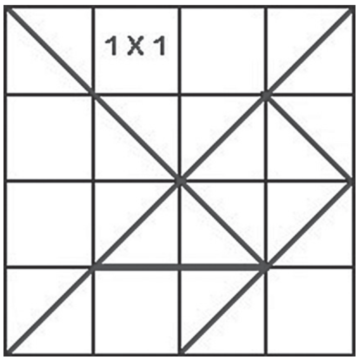


3. Rotation is a type of transformation where the original image is turned around a point, clockwise or anticlockwise.



Constructing Composite Shapes from Other 2D Shapes

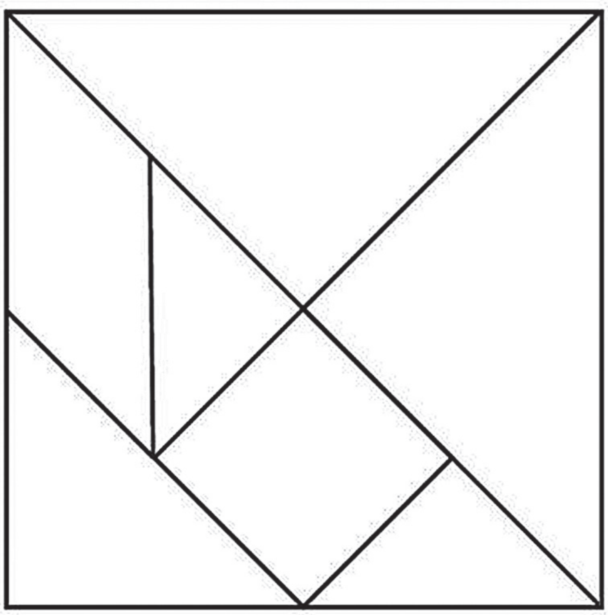
1. Learners start transforming shapes through tessellating the shapes of a tangram. They reflect, translate and rotate the shapes to create new composite shapes.

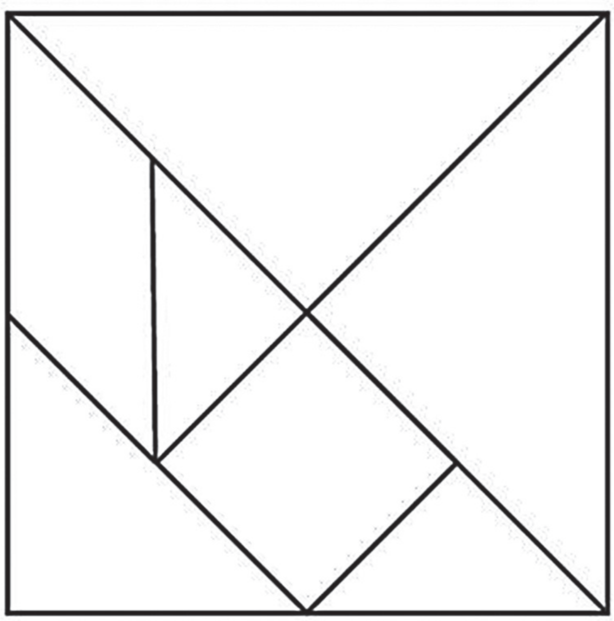


Construct Your Own Tangram

- Draw a four by four grid of exact squares as shown to the left.
- Mark off the lines as shown.
- Cut the shapes carefully along the lines.
- There are five triangles, one square and one parallelogram.

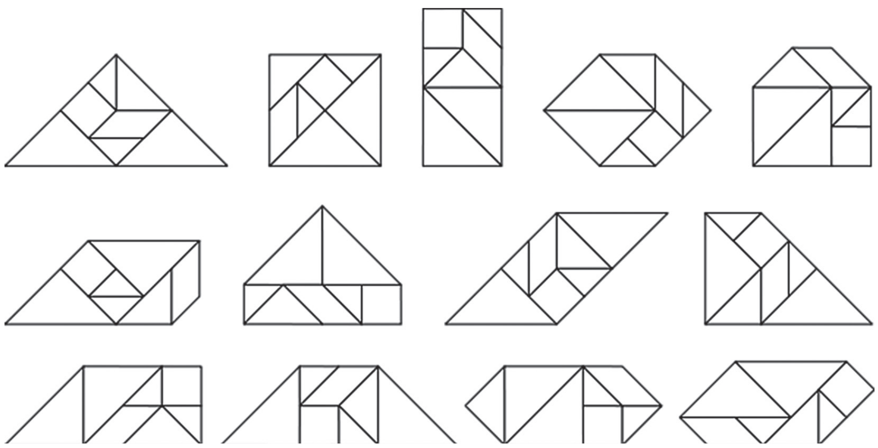
Cut your own tangram, or use the ones below:



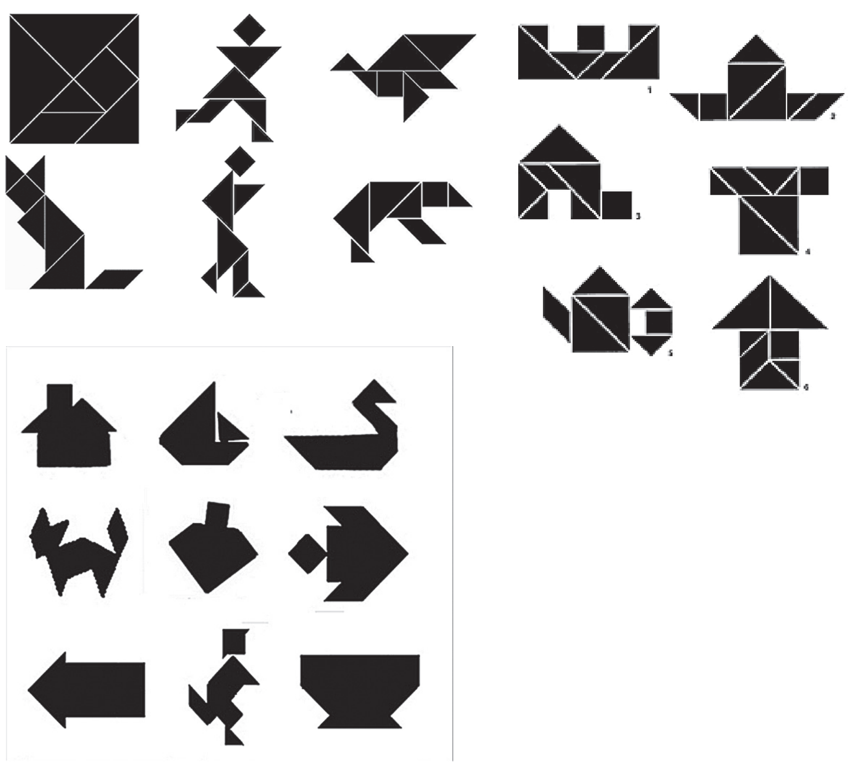


The rules for making new shapes with the seven shapes of a tangram

- All seven pieces must be used.
- All pieces must be put flat.
- Each piece must touch at least one other piece.
- No pieces may overlap.
- Pieces may be rotated or flipped.



Use these shapes to start making your own 2D shapes from the seven pieces of a tangram. After that, you may try some interesting other shapes like the ones below. (The first shapes show the small shapes, the second ones show only the big shape.)



Notes